A HYBRID FORM OF FUZZY-LOGIC CONTROLLER

Josef VRBA and Ywetta PUROVÁ

Institute of Chemical Process Fundamentals,
Czechoslovak Academy of Sciences, 165 02 Prague 6-Suchdol

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A hybrid form of a fuzzy- logic controller with linguistic and numerical variables and fuzzy subset as their common quantifier is presented. It is typical for the controller that the role of conditional statements of its linguistic model is optionally combined with a set of numerical parameters. The statements are automatically generated according to the given state of the system under control, it brings the necessary feature of adaptivity and self-tuning ability. The efficiency of the controller is checked on a set of simulated systems and a real one with pH control as well.

Some complex technological operations as, e.g., chemical batch reactors, cement kilns, wood dryers, etc. are difficult objects for the conventional automatic control. The difficulty is given, before all, by their highly non-linear and time-variable behaviour. Only such parameters are controlled, which may be easily measured and regulated, i.e. temperature, pressure, etc. The control of global parameters of the operation or process level as production quantity and quality rests usually upon the equipment operator. This fact is also the motive for the exploitation of the operator's intuition and experience for the construction of a control strategy.

An operator's control strategy is in its nature a set of heuristic resolutions,the description of which may be performed through the concept of linguistic variable. For the operator's resolution the uncertainty and vagueness of human reasoning, which is extremely unconvenient for numerical interpretation, is typical. He forms his resolution on a complex of measurements and observations (sometimes non--quantifiable ones as colour, taste, consistency, etc.). He categorizes them subjectively, a good deal of subjectivity is contained also in the deduced connections and relations. The set of heuristic (and also objectively valid) statements about rules of interference into the controlled process in given state forms the qualitative basis for a control strategy. The uncertain verabal statements can be converted into the desired numerical form by the aid of fuzzy mathematics.

The general opinion on fuzzy-logic control (FLC) may be expressed as follows:

1. The basic idea of FLC is to include the human experience into the controller's structure. The corresponding algorithm is constructed upon a set of heuristic rules

(statements), the verbal arguments of which are defined as fuzzy subsets (FSS). The main advantage of this concept is the possibility of exploitation of experience, intuition and the fact that the model of the controlled process is not necessary[1].

2. Some complex processes can be controlled by experienced operators with better results then with conventional controllers. The success corresponds with the qualitative nature of human reasoning and evaluation of non-quantifiable states of the process[2].

3. The direct digital control of technological processes needs a model description which is only a certain approximation of existing relations among real conditions. In case the process conditions change in wide range with significant stochastic disturbances, the control level does not correspond with expected results. The situation can be improved by the utilization of intuitive and experience-based strategy of a competent operator at least as a corrective factor everywhere the conventional tools and methods are improper[3].

4. The application of interval calculus in the concise form of FSS, by which it is possible to check the changes in the dynamics of the controlled process, is the significant advantage of FLC.

The construction of FLC is based on the following general tasks: (i) the determination of state and control variables of the system; (ii) the determination of the mode of representation of all quantities through FSS; (iii) the determination of linguistic rules for influencing the state of the system; (iv) the construction of corresponding algorithms for a necessary fuzzy arithmetics; (v) the selection of the mode of defuzzification of a fuzzy output for the necessary numerical form of the control action expected.

The General Form of FLC

The very base for any FLC is the fuzzy relation (FR) between the state of the controlled system $X \subset \mathscr{F}(R^n)$ and its control $U \subset \mathscr{F}(R^m)$. Both X and U are finite discrete universa with $n \times N_i$ elements $x_i^j \in X_i$ ($i = 1, \ldots, n \Leftrightarrow I_n$; $j = 1, \ldots, N_i \Leftrightarrow \Leftrightarrow J_{N_i}$) and $m \times M_k$ elements $u_k^p \in U_k$ ($k = 1, \ldots, m \Leftrightarrow K_m$; $p = 1, \ldots, M_k \Leftrightarrow P_{M_k}$) (the symbol $\Leftrightarrow$ serves for formal identity).

The total global FR (GFR) is constructed as the union

$$\mathscr{R} = \bigcup_{r=1}^{R} \mathscr{R}_r \tag{1}$$

of all GFR $\mathscr{R}_r$ ($r = 1, \ldots, R \Leftrightarrow R_r$) between fuzzy state $A_{i,r} \subset X_i$ and fuzzy control $B_{k,r} \subset U_k$ according to the conditioned linguistic statement "if A then B", i.e.

$$\bigcap_{i=1}^{n} A_{i,r} \rightarrow \bigcap_{k=1}^{m} B_{k,r}\,. \tag{2}$$

The membership functions (MF) of particular fuzzy quantities

$$\mu_{A_i} : X_i \rightarrow [0, 1] \tag{3}$$

$$\mu_{B_k} : U_k \rightarrow [0, 1] \tag{4}$$

$$\mu_{\mathcal{R}_r} : X \times U \rightarrow [0, 1] \tag{5}$$

are connected by the relation

$$\mu_{\mathcal{R}_r}(x, u) = \bigwedge_{I_n} \mu_{A_{i,r}}(x_i) \bigwedge_{K_m} \mu_{B_{k,r}}(u_k), \tag{6}$$

so that

$$\mu_{\mathcal{R}}(x, u) = \bigvee_{R_r} \mu_{\mathcal{R}_r}(x, u) \tag{7}$$

the symbols $\wedge$, $\vee$ stand for MIN, MAX, respectively).

The sought response B'_k for a concrete input $\bigcap_{i=1}^{n} A'_i$ (the symbols $\bigcap$, $\bigcup$ stand for intersection and union, respectively) is determined, e.g., for MAX–MIN composition with $\mathcal{R}$ by the formula

$$B'_k = A'_1 \circ (A'_2 \circ \ldots \circ (A'_n \circ \mathcal{R})) \tag{8}$$

with MF according to the algorithm

$$\begin{array}{l} \mu_{Y_0}(u_k) = \mu_{B_{k,r}}(u_k) \\ \hline \mu_{S_i}(x_i, u_k) = \mu_{A_{i,r}}(x_i) \wedge \mu_{Y_{i-1}}(u_k) \\ \quad \mu_{Y_i}(u_k) = \bigvee_{x_i} (\mu_{A_i'}(x_i) \wedge \mu_{S_i}(x_i, u_k)) \, ; \quad i = 1, \ldots, n \\ \hline \mu_{B'_{k,r}}(u_z) = \mu_{Y_n}(u_k) \, ; \quad r = 1, \ldots, R \\ \hline u_{B'}(u_k) = \bigvee_{r} \mu_{B'_{k,r}}(u_k) \end{array} \tag{A}$$

The natural claim for the construction of all evaluating term-sets is

$$\bigcup_{r=1}^{R} \mathrm{Supp}(A_{i,r}) = X_i \, ; \quad i = 1, \ldots, n \tag{9}$$

$$\bigcup_{r=1}^{R} \mathrm{Supp}\,(B_{k,r}) = U_k \, ; \quad k = 1, \ldots, m \, , \tag{10}$$

where Supp (C) is the support of the FSS C, i.e.

$$\mathrm{Supp}\,(C) = [t|\mu_C(t) > 0]\,. \tag{11}$$

The stronger claims

$$\forall i \in I_n, \forall j \in J_{N_i}, \exists r \in R_r : \mu_{A_{i,r}}(x_i^j) = 1 \tag{12}$$

$$\forall k \in K_m, \forall p \in P_{M_k}, \exists r \in R_r : \mu_{B_{k,r}}(u_k^p) = 1 \tag{13}$$

hang together with claims on the subtlety of determination of the controller's output or the interval of its insensitivity.

The Membership Function

For the description of MF as a function of an argument t, defined on its universe T, the parabolic formula[4]

$$\mu_C(t) = \begin{cases} 4\,\dfrac{(t - t^C_{\min})(t^C_{\max} - t)}{(t^C_{\max} - t^C_{\min})^2} & \text{for} \quad t^C_{\min} < t < t^C_{\max} \\ 0 & \text{otherwise} \end{cases} \tag{14}$$

the parameters $t^C_{\min}$, $t^C_{\max}$ of which limit at the same time Supp (C), $C \subset T$, is selected. From the point of the necessary evaluating term-sets and the determination of $\mu_{B'}(u_k)$ according to the algorithm (A), the formula (14) offers two principal advantages.

First, along with the selection of term-sets for evaluation of x and u it is possible to determine the particular values of MF with relatively high accuracy and according to our conceptions and aims. Moreover, the symmetry and limits of Supp (C) inform us directly about the accuracy of the linguistic model of the controller as to the extend of all universa and corresponding term-sets.

As $\mu_C(t)$ for $t \notin$ Supp (C) equals identically zero, it is possible to restrict the selection of statements for algorithm (A) in accordance with the composition (8) by the condition (15).

$$A'_i \cap A_{i,r} \neq 0\,! \tag{15}$$

The statements, not fulfilling the condition (15), may be omitted as those for given state inactive and thus the extend of logical and arithmetic operations may be reduced substantially. Indicating the parameters of MF (14) for x and u as $x^{i,r}_{\max}$, $x^{i,r}_{\min}$ and $u^{k,r}_{\min}$, $u^{k,r}_{\max}$ and the limits of Supp (A'_i) as $a^i_{\min}$, $a^i_{\max}$, the condition (15) can be reformulated into the form

$$(a^i_{\min} > x^{i,r}_{\max}) \cup (a^i_{\max} < x^{i,r}_{\min}) \overset{!}{=} \mathrm{FALSE}\,. \tag{16}$$

From practical considerations, the discretization of particular universa corresponds with the linear interpolation formula

$$t^{j} = t^{1} + \frac{j - 1}{N - 1}\,(t^{N} - t^{1})\,; \quad j = 1, \ldots, N \tag{17}$$

(where the indices 1 and N are for the first and last elements of T) and the conditions (*14*) and (*16*) can be rewritten into the index form, more convenient for computation. The reverse interpolation and projection (quantization) of A_i' into X_i is then made according to the general formula

$$j = \mathrm{INT}\left[1{\cdot}5 + \frac{t - x_i^j}{x_i^N - x_i^1}\,(N - 1)\right], \tag{18}$$

where INT[*] represents the integer part of *.

The Fuzzification of Input and the Defuzzification of Output of FLC

In principle, there are two possibilities how to evaluate the concrete state of the controlled system by FSS A_i'. It is either a vague, uncertain evaluation in accordance with the intuitive (mostly subjective) operator's conceptions as "high temperature" "low pressure", "middle concentration", etc., those being really the fuzzy values of variables x, or numerical equivalent of a real physical quantity directly measured on equipment. In both cases, however, the input information can be "digested" by FLC only if its numerical representation is in the prescribed accordance with the constructed rules of its model.

To begin with, any input into FLC must square with an element of the discrete net of X or U. Numerical quantities, not squaring with the selected numerical net, the controller cannot accept and work up. Second, any input information must be, even in the sense of linguistic description, unambiguous – the degree of its vagueness is from the formal point of view irrelevant.

The question is whether the operator is during his vague evaluation always able to define the corresponding values of MF on particular elements of their universe. That is why only three possibilities of input evaluation are considered:

1) uncertain evaluation according to the operator's own acceptably determined FSS,

2) uncertain evaluation according to accepted term-sets,

3) numerical evaluation according to the process data, properly projected on a given net of the universe under question.

In cases *1*) and *2*) the input evaluation is given by Supp (A_i') with limits $N_i^1, N_i^2 \in$ $\in J_{N_i}$. In case *2*) the natural condition

$$\exists r \in R_r : (N_i^1 \equiv N_{i,r}^1) \& (N_i^2 \equiv N_{i,r}^2) \qquad (19)$$

is valid, while in case *3*) $a_{\min}^i = a_{\max}^i = a^i$ and, according to Eq. (*18*),

$$\mu_{A'_1}^j = \begin{cases} 1 & \text{for} \quad j = j_{a^i} \\ 0 & \text{otherwise} . \end{cases} \qquad (20)$$

For output defuzzification three possibilities are offered:

1) the comparison of an unsharp output with accepted term-sets with the aid of MF value for all discrete elements of u,

2) the comparison of the peak of an unsharp output's MF with the positions of peaks of MF of corresponding term-sets,

3) the comparison of the peak of an unsharp output's MF with the positions of discrete elements of the corresponding universe.

In connection with all presented possibilities of defuzzification, it is necessary to mention a certain phenomena often accompanying the confrontation of an unsharp output with possible tools of its sharp evaluation. It is the problem of polymodality of an output's MF which does not mind in case *1*) but brings serious difficulty in cases *2*) and *3*). In the literature on fuzzy control the problem of polymodality is usually solved through the mean value of the position of all distinct peaks of the output MF. Such an approach is, however, in sharp contradiction with the very nature of the notion of FSS. Moreover, it contradicts also its MF as an expedient of description and measure of uncertainty.

The basic role of MF is in the amount of unsharp restriction put on the membership of an element to FSS. The element can be the more included into a given FSS, the higher is its degree of membership to it. And just here is the weakest point of the mentioned way of defuzzification of the polymodal FSS. The mean value of positions of two neighbouring peaks must quite logically correspond with the position of the saddle formed by those peaks and thus with the element of a relatively maximum uncertainty. That is, however, to the very contrary of what we aim to reach by defuzzification.

There are practically only two reasons of polymodality in the output MF. The first one is quite objective and given by the character of the controlled system. The second one is subjectively connected with the existency of two or more ambiguous statements about the functioning of the modelled and controlled system.

The experience shows that the most reliable way to protect the output from polymodality is to analyse the set of statements from the point of their unambiguity as well as the a posteriori comparison of the output FSS with all evaluating term-sets. This way is in details discussed elsewhere[5], it is worth mentioning here that the notion of Hamming distance[6] serves quite well for this purpose.

The SISO FLC

When using a difference form of FLC with single input and single output (SISO)

$$u(t) = f((x(t - \tau_x) - \mathrm{SP}(t - \tau_x))) \qquad (21)$$

(where SP is the symbol of set-point and τ_x the time delay in control) and sufficiently small spreads $(x^N - x^1)$ and $(u^M - u^1)$, it is possible to expect the MF of the output FSS B' in a unimodal form (maximally with a plateau). It may be useful then to include the defuzzification of B' directly into the construction of the corresponding fuzzy relation and any statement of the model formulate as

$$A_r \rightarrow u_{p_{or}}\,; \quad p_{or} \in P_M, r \in R_r\,, \qquad (22)$$

and to define the MF of the term-set B_r as

$$\mu_{B_r}(u^p) = \begin{cases} 1 & \text{for} \quad p = p_{or} \\ 0 & \text{otherwise}\,. \end{cases} \qquad (23)$$

The superscript p_{or} corresponds to the presumption of the position of MF's peak on U. Then, of course,

$$\mu_{\mathscr{R}_r}(x^j, u^p) = \begin{cases} \mu_{A_r}(x^j) & \text{for} \quad p = p_{or} \\ 0 & \text{otherwise}\,. \end{cases} \qquad (24)$$

Without any loss of generality, the relation between the indices r and p_{or} may be for the acceptable presumption

$$N = M = R\,; \quad j, p, r \in R_r \qquad (25)$$

expressed by the formula

$$p_{or} = R - r + 1\,. \qquad (26)$$

In automatic control, any input A' is FSS in the sense of Eq. (*20*). It is possible then, to formulate the statement (*22*) in the form

$$x_r \rightarrow u_{p_{or}} \qquad (27)$$

and the MF of A_r define as

$$\mu_{A_r}(x^j) = \begin{cases} 1 & \text{for} \quad j = r \\ 0 & \text{otherwise}\,. \end{cases} \qquad (28)$$

Because (according to Eqs (*6*), (*23*), and (*26*)) the formula

$$\mu_{\mathscr{R}_r}(x^j, u^p) = \begin{cases} 1 & \text{for } j = r \text{ and } p = R - r + 1 \\ 0 & \text{otherwise} \end{cases} \qquad (29)$$

holds, the model of the SISO FLC is based upon the relation

$$\mathscr{R} \equiv I^{-1}, \qquad (30)$$

where I^{-1} is the result of the quadrant rotation of the crisp equivalence relation I. The composition (*8*), i.e.

$$B' = A' \circ I^{-1} \qquad (31)$$

is apparently identical with the relation (*26*).

The formulae (*27*), (*29*), and (*30*) express the relation between the indices of numerical representatives of term-sets on X and U, by which the unsharp relation $A' \to B'$ is evaluated through the accepted form of the relation $\mathscr{R}$ as the basis of the FLC. They represent the relation between the interval values on X and U, this fact supports also the idea that the fundamental form of FLC corresponds with that of a multilevel relais[7].

The real error $(\mathrm{d}_x)_s = (x\text{-SP})$ is projected through the relation (*18*) (where $(\mathrm{d}x)_s$ stands for t, $\mathrm{d}x^1$ for x^1 and $\mathrm{d}x^N$ for x^N, the last two errors being the limits of the corresponding universe X with N elements) on the value $\mathrm{d}x^r \in X$. This value evokes the control action according to the scheme

$$(\mathrm{d}x)_s \to [\mathrm{d}x^r \in X] \to [\mathrm{d}u^{\mathrm{Por}} \in U] \qquad (32)$$

expressing the transfer of $(\mathrm{d}x)_s$ as an element of a *continuous* numerical axis on the value $\mathrm{d}u^{\mathrm{Por}}$ as an element of a *discrete* universe U. The first phase of this transfer corresponds with the fuzzification (quantisation) of the crisp input, the second one with the mapping of the quantised input on the quantised output. The accuracy and sensitivity of the resulting control action of the mentioned basic form of the SISO FLC would be determined by the level of discretisation of X.

The finer is the net of X, the finer may be expected the discrimination of particular elements of U. But the level of discretisation is conditioned by the level of uncertainty of the controlled system and its connections with the environment. It is rather impossible to increase the number of elements of X above the level of their minimum possibility of discrimination, i.e. above the level when FLC loses its very purpose and sense. On the other hand, the level of quantisation of $\mathrm{d}u$ determines the quality of the control process, first of all the interval of insensitivity of FLC to the behaviour of the controlled object. This problem fully rests upon the user's opinion and has the form of the third phase of the schema (*32*): du^{Por} is to be transferred into a "really useful" value $(du)_s$.

The notion of a "really useful" value is the vague notion which, however, corresponds in some way with the minimization of an error bellow the acceptable level of quantisation of X and U. The natural way of the third phase of the scheme (*32*) is, e.g., the proportion

$$(du)_s \sim du^{Por} \,.\, |(dx)_s| \qquad (33)$$

from which logically results that

$$\lim_{|(dx)_s| \to 0} (du)_s = 0 \,. \qquad (34)$$

The final form of the transfer (*33*) is determined by the general claims on the quality of the control process from the point of SP and its variability.

In the presented version of the SISO FLC the concrete form of the relation (*33*) is

$$(du)_s = (1 + \alpha)(1 - aK_1)(1 - 2K_2) \,.\, |(dx)_s| \,.\, du^{Por} \,, \qquad (35)$$

where α is acceleration parameter, K_1 binary 0/1 constant respecting the fact of attaining the zone of control process stability, K_2 binary 0/1 constant respecting the fact of attaining the estimated zone of control process damping, a empirical constant with the recommended value 0·7 (this recommendation is based upon the author's experience).

Acceleration parameter α is set up so to attain the desired state of the system from any other one as soon as possible. Its value is determined experimentally. The binary constant K_1 has to satisfy the condition

$$[|(dx)_s^k| < K_1 b + (1 - K_1)\beta|SP - x^0|] \text{ and}$$
$$\text{and } [(dx)_s^k((dx)_s^k - (dx)_s^{k-1}) < 0] \,, \qquad (36)$$

where x^0 is starting state of the system before its change, b is empirical constant characterizing the estimated zone of damping (the recommended value is 0·3% of the interval of possible values of state of the system) and k is time index of sampling.

If the condition (*36*) is fulfilled, then $K_2 = 1$ – otherwise $K_2 = 0$. Apparently, $K_2 = 0$ characterizes the acceleration phase when $(1-2K_2) = +1$, while $K_2 = 1$ is for the damping phase when $(1-2K_2) = -1$. The value of β (damping parameter) is determined experimentally so to minimize the possible overshoot and oscillations around the desired SP. The attaining of the zone of the control stability corresponds to the condition

$$(\mathrm{d}x)_s^k (\mathrm{d}x)_s^{k-1} \leqq 0 . \qquad (37)$$

The zone itself is limited by the interval $(\mathrm{SP} \pm \beta|\mathrm{SP} - x^0|)$. $K_1 = 1$ if the condition (*37*) is fullfiled, otherwise $K_1 = 0$. With $K_1 = 1$ also the additive damping influence of the term $(1 - aK_1)$ on the accelerating term $(1 + \alpha)$, the value of which is thus decreasing in its power within the zone of stability, is asserted. Moreover, the value of the control variable u can be restricted (along with fulfilling the condition (*37*)) by the limits u_{min}, u_{max}, being in a relation with u^1, u^M as boundary points of U:

$$u_{\mathrm{min}} = [1 - \mathrm{sign}(\gamma) \, . \, b] \, \gamma \, \mathrm{SP} \geqq u^1 \qquad (38)$$

$$u_{\mathrm{max}} = [1 + \mathrm{sign}(\gamma) \, . \, b] \, \gamma \, \mathrm{SP} \leqq u^M . \qquad (39)$$

The parameter γ represents the experimental evaluation of the static gain of the controlled system. Then

$$(u)^k = (u)^{k-1} + (\mathrm{d}u)_s^k \qquad (40)$$

and

$$u_{\mathrm{min}} \leqq (u)^k \leqq u_{\mathrm{max}} . \qquad (41)$$

As for the universa and term-sets of x and u, the basic information concentrates around the relation between $\mathrm{d}x$ and $\mathrm{d}u$. The values of $\mathrm{d}x^1$, $\mathrm{d}x^N$ and $\mathrm{d}u^1$, $\mathrm{d}u^M$ are derived from the starting data on x^1, x^N and u^1, u^M according to the formulae

$$\mathrm{d}x^N = [\mathrm{MAX}((\mathrm{SP} - x^1), \; (x^N - \mathrm{SP}))]/10 \qquad (42)$$

$$\mathrm{d}x^1 = -\mathrm{d}x^N \qquad (43)$$

$$\mathrm{d}u^M = \mathrm{sign}(\gamma) \, |u^M - u^1|/100 \qquad (44)$$

$$\mathrm{d}u^1 = -\mathrm{d}u^M . \qquad (45)$$

We consider uniquely 9 elements and (hypothetically) 9 term-sets for both variables, i.e. $N = M = 9$. These values are of empirical character and have been proved experimentally as optimal, the same is with the quantities 10, and 100 in Eqs (*42*) and (*44*), respectively.

Of course, the question of the full utilization of the role of the model (*35*) or the simple satisfaction with the fuzzy-linguistic form (*32*) depends on the character of the object under control. That is what we mean by the notion "hybrid form" of FLC. Both forms may be considered as limits for fuzzy and/or conventional digital control, the role of the parameters α, β and γ may be then either enforced or suppresed. In any case, we are interested in possible way of their identification.

It is possible to determine the values of α, β and γ empirically from the response curve on the unit change of x. The procedure itself does not represent a special problem, the value of γ is given by the limit value of the ratio u/x (static gain) and α, β are determined simultaneously from the response with respect to the maximally attainable quality of the control process. Examples of the identification course are presented in Figs 1 and 2.

The usual way is to make the first 50–100 sampling steps of the control process with zero values of the identified parameters. This is sufficient for evaluation of γ and the first approximations of α and β. The controller follows automatically the

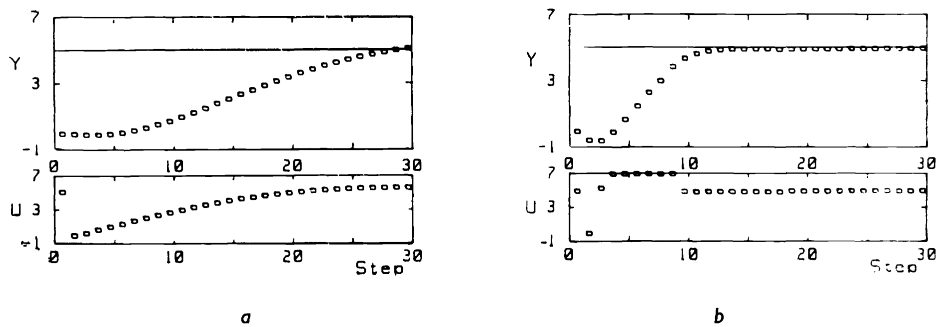

FIG. 1

Identification of FLC parameters. *a* $x(t) - 1{\cdot}425x(t-1) + 0{\cdot}496x(t-2) = -0{\cdot}102u(t-1) + 0{\cdot}173u(t-2)$; $\alpha = 0$, $\beta = 0$, $\gamma = 0$. *b* $x(t) - 1{\cdot}425x(t-1) + 0{\cdot}496x(t-2) = -0{\cdot}102u(t-1) + 0{\cdot}173u(t-2)$; $\alpha = 0{\cdot}4$, $\beta = 14{\cdot}2$, $\gamma = 1{\cdot}0$

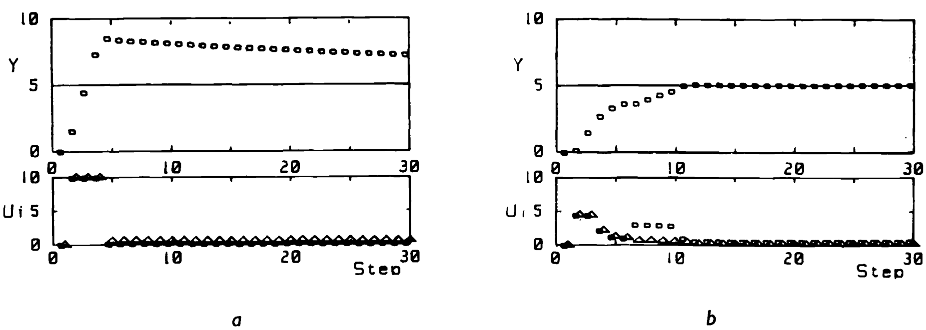

FIG. 2

Identification of FLC parameters. *a* $x(t) - 0{\cdot}948x(t-1) + 0{\cdot}001x(t-2) = -0{\cdot}102u_1(t-1) + 0{\cdot}173u_1(t-2) + 0{\cdot}138u_2(t-1) + 0{\cdot}088u_2(t-2)$; $\alpha = 0$, $\beta = 0$, $\gamma = 0$. *b* $x(t) - 0{\cdot}948x\,.\,(t-1) + 0{\cdot}001x(t-2) = -0{\cdot}102u_1(t-1) + 0{\cdot}173u_1(t-2) + 0{\cdot}138u_2(t-1) + 0{\cdot}088u_2\,.\,(t-2)$; $\alpha = -0{\cdot}7$, $\beta = 0$, $\gamma_1 = 0{\cdot}11$, $\gamma_2 = 0{\cdot}03$

development of the mean value of the ratio u/x within the area limited by $K_1 = 1$, i.e. within the zone of control stability. The influence of the parameter β is quantised on the intervals of the error (x-SP), the estimation of its value is thus simpler. It is possible to recommend the way in which the quantity $\beta(\mathrm{SP} - x^0)$ approximately equals one half of the overshoot amplitude at zero values of α and γ.

For starting approximations of α and β, the empirical formulas

$$\alpha = |\gamma| \, \delta/\mathrm{d}u_{\max} \tag{46}$$

$$\beta = \delta/(2|\mathrm{SP} - x^0|)\,, \tag{47}$$

where $\mathrm{d}u_{\max}$ is maximum absolute increase in the value of the control variable at the minimum $x_{\min}$, or maximum $x_{\max}$ value of the state variable,

$$\delta = (1 - m)\, x_{\min} + m x_{\max} - \mathrm{SP} \tag{47a}$$

and

$$m = \begin{cases} 0 & \text{for} \quad x^0 > \mathrm{SP} \\ 1 & \text{for} \quad x^0 < \mathrm{SP} \end{cases} \tag{48}$$

have been checked as quite satisfactory.

The inaccurate estimation of the parameter γ, or the time change in the behaviour of the controlled system is faced by the adaptive restriction of the control variable in the form

$$\gamma_{\text{new}} = \gamma_{\text{old}} - \operatorname{sign}(\gamma)\,|(u_{\max} - u_{\min})/(\mathrm{SP} - x^0)| \quad \text{for} \quad u = u_{\min} \tag{49}$$

$$\gamma_{\text{new}} = \gamma_{\text{old}} + \operatorname{sign}(\gamma)\,|(u_{\max} - u_{\min})/(\mathrm{SP} - x^0)| \quad \text{for} \quad u = u_{\max}\,. \tag{50}$$

The new restriction $u_{\min}$ and $u_{\max}$ for γ_{new} are adapted according to Eqs (*38*) and (*39*).

In majority of systems, having served as test problems, the first approximations of α, β, γ were accepted also as the correct ones. Some systems, however, exhibited a strong tendency for divergency or at least for distinct oscillations around SP. In these cases the most efficient way for the parameters identification was the succesive approximation of β till the suppresing of the mentioned oscillations (if possible) with the following estimation of α and finally of γ.

The MISO FLC

Another version of FLC presented here is its multiple input/single output(MISO) form which works with the values of x and u (instead of their increments $\mathrm{d}x$ and $\mathrm{d}u$)

on the basis of a linguistic model. The simplest version of the model utilizes only two statements respecting the positive and negative changes in variable's values. The increase in sensitivity of the controller claims, however, higher number of evaluating terms and more elements in universa, i.e. higher distinguishability of states of the controlled system. This fact brings also a higher need in computing time and processor's memory. Apparently, these two points of view demand a certain compromise[8].

Up to now presented versions of FLC work with a beforehand constructed set of statements, it is not the case of what is presented here. The controller works with an automatic generation of particular statements of the linguistic model. That means, that from the very beginning of its controlling activity, the controller constructs the set of statements for all the up-to-now known quantised states of the system. About the orientation of influence of particular control variables decide their static gain orientation (known in advance) and the sign of control error and its magnitude. All variables of the system are quantised on a given net of universa, the quantised values are then confronted with corresponding term-sets, and the result in some ordered sequence forms the proposition of the conditioned statement. All generated statements are continually checked during the control process and efficiently reduced or extended in their number according to the quality of the control. The formal rule of term-sets ordering in the statement is similar to that of Eq. (*26*).

As in the SISO FLC, the parameters α, β, γ_i, the last of which being n-dimensional vector, are employed to improve the control activity of FLC. All control variables act simultaneously according to the linguistic model and the information on control error.

The Application of FLC *to the Control of Simulated and Real Systems*

To check the efficiency of the constructed FLC forms, a lot of simulation and practical experiments on a real object were made using the standard form of digital PID controller and the selftuning receding horizon controller with variable forgetting factor RECEX for comparison. The latter uses the square root identification procedure. The brief presentation of both controllers may be found, e.g., in the paper[9].

Typical dynamic models in the form of difference equation were selected for the simulation. These models are usually presented as

$$x(k) - \sum_{j=1}^{t} a_j x(k-j) = \sum_{i=1}^{n} \sum_{j=1}^{t} b_{ij} u_i(k-s-j+1)\,, \tag{51}$$

where $s \geqq 1$ is time delay and $t \geqq 1$ is model order. Z-transform of Eq. (*51*) gives

$$X(z)\,A(z^{-1}) = U(z)\,B(z^{-1})\,. \tag{52}$$

The properties of the model (5*1*) are determined by the position of roots of the polynomials $A(z^{-1})$ and $B(z^{-1})$ in complex plane. The presented work is concentrated first of all on minimum phase and stability property as those of special importance. A system is understood as minimum phase if all roots of $B(z^{-1})$ (zeros) are inside the unit circle. If, moreover, all roots of $A(z^{-1})$ (poles) are inside the unit circle the system is stable.

Digitally controlled continuous-flow reactor (Fig. 3) is equipped with sensors and actuators to enable the control of pH (among other state variables). Both the acid and alkalic component are dosed from corresponding reservoirs through digitally controlled peristaltic pumps. Simultaneously, water as inert flows continuously at constant or variable rate into the reactor from normal water supply. pH is measured through the analog output signal by pH-meter connected with the computer. The mixer is driven by a continuously controlled engine.

The simulation experiments with the models (5*1*) were divided into three groups:

1. control parameters setting for given time-independent dynamics,
2. no exact correspondence between parameters and dynamics of the system,
3. modification of experiments ad 2 by a random noise superposition on both input and output signals.

Taking into account the reality of process control, the variable U is always constrained. The constraints need not be necessarily always active but for systems of integral behaviour may become the factor of importance. The detailed discussion of the simulation results may be found in the mentioned paper[9], here we summarize the global view on applicability of FLC in the comparison with PID and RECEX controllers.

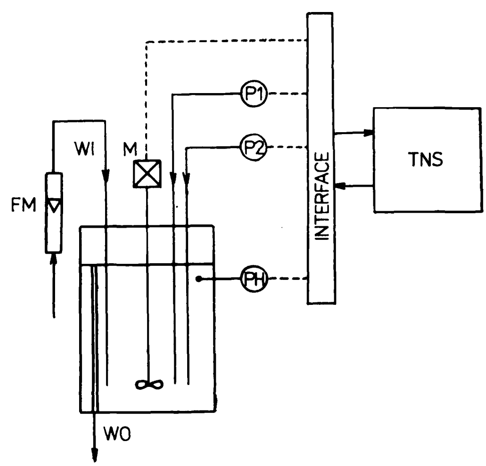

FIG. 3
Scheme of the experimental equipment for pH-control: P1 alkali pump, P2 acid pump, WI water inlet, WO water outlet, PH pH-meter, M mixer, FM flow-meter

Because of the integral behaviour of astatic systems (poles and zeros are close to unity) the tuning of FLC and PID parameters must have been done empirically, the identification of RECEX were rather unstable too. The best control results were obtained with PID controller.

For systems with unsteady dynamic behaviour, however, the more sophisticated control algorithms of FLC and RECEX dominated. FLC parameters tuning suffered a little from the nonstability of the system. Better control results with FLC and RECEX were also obtained for minimum phase and stable systems.

The systems without significant changes in their characteristics can be controlled fairly well by the digital version of PID algorithm. The systems with variable dynamics, however, demand more sophisticated approach with adaptive and selftuning possibilities. FLC seems satisfactory in this sense because of its smaller sensitivity to any type of change in the behaviour of the controlled system and to its adaptivity feature due to the automatic generation and continuous correction of linguistic statements of the model.

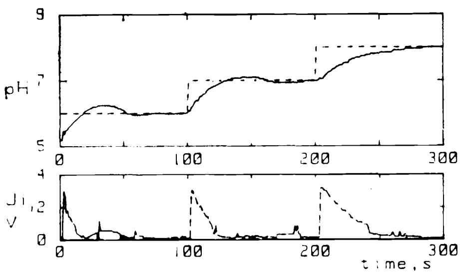

FIG. 4

pH control with FLC on set-points; – – – set-points, ——— pH measured; ——— acid pump, –·–·– alkali pump

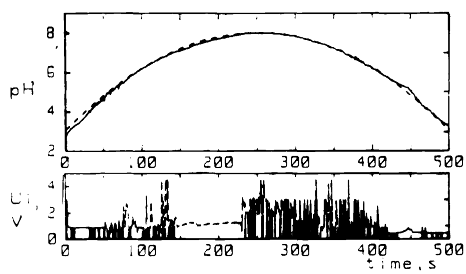

FIG. 5

pH control with FLC on trajectory; – – – set-points, ——— pH measured; ——— acid pump, –·–·– alkali pump

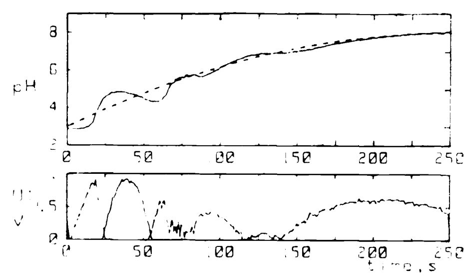

FIG. 6

pH control with PID on trajectory; – – – set-points, ——— pH measured; ——— acid pump, –·–·– alkali pump

Interesting are the results of the application of FLC to the control of pH in the experimental equipment described above. To evaluate the behaviour of the controller within the whole area of measurable pH values, the experiments include before all the set of succesive unit changes of set-points and the control on non-linear trajectory as well. Some typical results are illustratively summarized in Figs 4–6, the last of which describes the comparative behaviour of the tuned conventional PID controller. As seen, the system is taken as that of 1^{st} order with two inputs and one output, the inputs beeing the activities of both peristaltic pumps within the intervals 0–5 Volt continually measured and regulated through the TNS microcomputer (64 kBytes).

From the three parameters α, β, γ only the damping parameter β plays its role, the remaining two have not been taken into account, i.e. $\alpha = \gamma = 0$. According to the titration curve of the acid/alkali system the damping parameter has to be formulated in a relation with the pH level. The formula

$$\beta = 0{\cdot}8\,(1 - 0{\cdot}1\ \mathrm{SP}) \qquad (53)$$

serves for this purpose within the interval of pH 2–10. The other values havbeen out of practical interest due to the character of the system.

An important role for the FLC identification plays also the relation among sam. pling interval and the subtlety of universes and term-sets of the controller's variables As mentioned above, the finer is the net of X, the finer may be expected the discrimination of particular elements of U. But the increase in number of elements in X sharply contradicts with the memory possibilities of the computer and some compromise is thus needed. The experiments proved the optimum relation 30 elements for the universe and evaluating term-scale for every variable and the sampling interval 10 seconds. More elements bring serious problems in performing the necessary arithmetic and logical operations of the FLC within the given sampling interval, further enlargement of which is inefficient. Under these conditions, the number of automatically generated model statements is usually 4 for separately given set-point and approximately 20 for the control on a trajectory. It is necessary to emphasize that the increasing number of statements brings also longer time for all arithmetic and logical operations performed in the frame of the FLC model.

CONCLUSION

However the character of FLC predestines it to the exploitation for so called "global" control of processes including their diagnostics and long-time analysis, FLC is far not inferior in comparison with conventional controllers. The automatic generation of model statements enables it to face the changes in dynamics of the system under control and brings thus the necessary adaptive features, typical for the more sophisticated adaptive and self-tuning controllers. The efficiency of the presented FLC has been thus checked first of all from the point of conventional regulation either simulated or a real one. It may be concluded that the behaviour of the FLC is satisfactory even in this sense.

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